APRIL/MAY 2023

DMA43 — FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

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1. Define Banach space

2. Define linear transformation.

3. Define orthonormal.

4. Give the properties of orthogonal complements.

5. Write unitary operator.

6. Define conjugate space.

7. Give the formula for spectral radius.

8. Define Banach algebra.

9. Write commutative property.

10. Define Gelfand mapping.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove Minkowski's inequality.

Or

(b) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, prove that there exists a functional f_0 on N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.

12. (a) State and prove uniform boundedness theorem.

Or

(b) State and prove open mapping theorem.

13. (a) Show that the unitary operators on H form a group.

Or

(b) If P is the projection on a closed linear subspace M of H, prove that M is invariant under an operator $T \Leftrightarrow TP = PTP$.

14. (a) Prove that the boundary of S is a subset of Z.

Or

(b) If O is the only topological divisor of zero in A, prove that $A = C$.

15. (a) If f_1 and f_2 are muliplicative functionals on A with the same null space M, prove that $f_1 = f_2$.

Or

(b) If A is self-adjoined, prove that $\hat{A}$ is dense in $\mathcal{C}(\mathfrak{M})$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Hann — Banach Theorem.

17. A non empty subset X of a normed linear space N is bounded $\Leftrightarrow f(X)$ is bounded set of numbers for each f in N^*.

18. Let H be a Hilbert space, and let f be an arbitrary functional in H^*. Prove that exits a unique vector y in H such that $f(x) = (x,y)$ for every x in H.

19. Show that $r(x) = \lim \|x^n\|^{\frac{1}{n}}$.

20. State and prove Gelfand – Neumark Theorem.
